END 331E - OPERATIONS RESEARCH I

HOMEWORK 2 - SOLUTIONS

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Question 1

a)

a)	**1**	**b)**	**0**
c)	**0**	**d)**	**0.5**
e)	**0**	**f)**	**1.5**
g)	**0**	**h)**	**0**
i)	**11.5**	**j)**	**5.5**
k)	**1.5**	**l)**	**3**
m)	**5.5**	**n)**	**3**

b.) max z= x1+2x2 (objective function)
Constraints:
2x1+x2<=14
x1+x2<=10
x2<=3
x1>=0
x2>=0

c.) For the constraint x2<=3, that binds x2, we can descrease 3 units that we satisfy positivity constraint. Additionally, we can increase 3 units tha does not change the the basic variables (1.5x2=3)

Question 2

Standard form is obtained after adding x_4, x_5 and x_6 as slack variables and x_7 as excess variable.

Standard form

$$
\begin{aligned}
\max z = \quad & x_1 - 2x_2 + x_3 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 = 6 \\
& 2x_1 + x_2 + x_5 = 5 \\
& -x_1 + 2x_2 - x_3 + x_6 = 4 \\
& x_1 + x_2 - \mathrm{x}_7 = 1 \\
& x_2 + x_3 = 4 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0
\end{aligned}
$$

The modified objective function is obtained after adding artificial variables $x_8,\ x_9$.

$$z_{big-M} = x_1 - 2x_2 + x_3 - Mx_8 - Mx_9$$

$$\begin{aligned}
\max z_{big-M} = \quad & x_1 - 2x_2 + x_3 - Mx_8 - Mx_9 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 = 6 \\
& 2x_1 + x_2 + x_5 = 5 \\
& -x_1 + 2x_2 - x_3 + x_6 = 4 \\
& x_1 + x_2 - x_7 + x_8 = 1 \\
& x_2 + x_3 + x_9 = 4 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0
\end{aligned}$$

Initial Table

z_{big-M}	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	RHS
1	-1	2	-1	0	0	0	0	M	M	0
0	1	1	1	1	0	0	0	0	0	6
0	2	1	0	0	1	0	0	0	0	5
0	-1	2	-1	0	0	1	0	0	0	4
0	1	1	0	0	0	0	-1	1	0	1
0	0	1	1	0	0	0	0	0	1	4

Initial Table (Appropriate form)

	z_{big-M}	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	RHS
	1	-M-1	-2M+2	-M-1	0	0	0	M	0	0	-5M
x_4	0	1	1	1	1	0	0	0	0	0	6
x_5	0	2	1	0	0	1	0	0	0	0	5
x_6	0	-1	2	-1	0	0	1	0	0	0	4
x_8	0	1	1	0	0	0	0	-1	1	0	1
x_9	0	0	1	1	0	0	0	0	0	1	4

<u>Iteration 1</u>

	BASIS INVERSE					RHS
z_{big-M}	0	0	0	0	0	-5M
x_4	1	0	0	0	0	6
x_5	0	1	0	0	0	5
x_6	0	0	1	0	0	4
x_8	0	0	0	1	0	1
x_9	0	0	0	0	1	4

x_2 enters the basis

$$\mathbf{y_2} = \mathbf{B^{-1}a_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} z_2 - c_2 \\ \mathbf{y_2} \end{bmatrix} = \begin{bmatrix} -2M + 2 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

	BASIS INVERSE					RHS	x_2	Ratio
z_{big-M}	0	0	0	0	0	-5M	-2M+2	
x_4	1	0	0	0	0	6	1	6
x_5	0	1	0	0	0	5	1	5
x_6	0	0	1	0	0	4	2	2
x_8	0	0	0	1	0	1	1	1
x_9	0	0	0	0	1	4	1	4

x_8 leaves the basis

Iteration 2

	BASIS INVERSE					RHS
z_{big-M}	0	0	0	2M-2	0	-3M-2
x_4	1	0	0	-1	0	5
x_5	0	1	0	-1	0	4
x_6	0	0	1	-2	0	2
x_2	0	0	0	1	0	1
x_9	0	0	0	-1	1	3

$\mathbf{w} = [0 \quad 0 \quad 0 \quad 2M-2 \quad 0]$

For non-basic variables calculate $z_j - c_j = \mathbf{wa_j} - c_j$

$$z_1 - c_1 = \mathbf{wa_1} - c_1 = [0 \quad 0 \quad 0 \quad 2M-2 \quad 0] \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} - (M+1) = M - 3$$

$$z_3 - c_3 = \mathbf{wa_3} - c_3 = [0 \quad 0 \quad 0 \quad 2M-2 \quad 0] \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} - (M+1) = -M - 1$$

$$z_7 - c_7 = \mathbf{wa_7} - c_7 = [0 \quad 0 \quad 0 \quad 2M-2 \quad 0] \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} - (-M) = -M + 2$$

$$z_8 - c_8 = \mathbf{wa_8} - c_8 = [0 \quad 0 \quad 0 \quad 2M-2 \quad 0]\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} - 0 = 2M - 2$$

x_3 enters the basis

$$\mathbf{y_3} = \mathbf{B^{-1}a_3} = \begin{bmatrix}1 & 0 & 0 & -1 & 0\\0 & 1 & 0 & -1 & 0\\0 & 0 & 1 & -2 & 0\\0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & -1 & 1\end{bmatrix}\begin{bmatrix}1\\0\\-1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\0\\-1\\0\\1\end{bmatrix} \qquad \begin{bmatrix}z_3 - c_3\\ \mathbf{y_3}\end{bmatrix} = \begin{bmatrix}-M-1\\1\\0\\-1\\0\\1\end{bmatrix}$$

	BASIS INVERSE					RHS	x_3	Ratio
z_{big-M}	0	0	0	2M-2	0	-3M-2	-M-1	
x_4	1	0	0	-1	0	5	1	5
x_5	0	1	0	-1	0	4	0	-
x_6	0	0	1	-2	0	2	-1	-
x_2	0	0	0	1	0	1	0	-
x_9	0	0	0	-1	1	3	1	3

x_9 leaves the basis

<u>Iteration 3</u>

	BASIS INVERSE					RHS
z_{big-M}	0	0	0	M-3	M+1	1
x_4	1	0	0	0	-1	2
x_5	0	1	0	-1	0	4
x_6	0	0	1	-3	1	5
x_2	0	0	0	1	0	1
x_3	0	0	0	-1	1	3

$\mathbf{w} = [0 \quad 0 \quad 0 \quad M-3 \quad M+1]$

For non-basic variables calculate $z_j - c_j = \mathbf{wa_j} - c_j$

$$z_1 - c_1 = \mathbf{wa_1} - c_1 = [0 \quad 0 \quad 0 \quad M-3 \quad M+1]\begin{bmatrix}1\\2\\-1\\1\\0\end{bmatrix} - (M+1) = -4$$

$$z_7 - c_7 = \mathbf{wa_7} - c_7 = \begin{bmatrix} 0 & 0 & 0 & M-3 & M+1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} - (-M) = 3$$

$$z_8 - c_8 = \mathbf{wa_8} - c_8 = \begin{bmatrix} 0 & 0 & 0 & M-3 & M+1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - 0 = M - 3$$

$$z_9 - c_9 = \mathbf{wa_9} - c_9 = \begin{bmatrix} 0 & 0 & 0 & M-3 & M+1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - 0 = M + 1$$

x_1 enters the basis

$$\mathbf{y_1} = \mathbf{B^{-1}a_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -4 \\ 1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} z_1 - c_1 \\ \mathbf{y_1} \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 1 \\ -4 \\ 1 \\ -1 \end{bmatrix}$$

	BASIS INVERSE					RHS	x_1	Ratio
z_{big-M}	0	0	0	M-3	M+1	1	-4	
x_4	1	0	0	0	-1	2	1	2
x_5	0	1	0	-1	0	4	1	4
x_6	0	0	1	-3	1	5	-4	-
x_2	0	0	0	1	0	1	1	1
x_3	0	0	0	-1	1	3	-1	-

x_2 leaves the basis

Iteration 4

	BASIS INVERSE					RHS
z_{big-M}	0	0	0	M+1	M+1	5
x_4	1	0	0	-1	-1	1
x_5	0	1	0	-2	0	3
x_6	0	0	1	1	1	9
x_1	0	0	0	1	0	1
x_3	0	0	0	0	1	4

$$\mathbf{w} = [0 \quad 0 \quad 0 \quad M+1 \quad M+1]$$

For non-basic variables calculate $z_j - c_j = \mathbf{wa_j} - c_j$

$$z_2 - c_2 = \mathbf{wa_2} - c_2 = [0 \quad 0 \quad 0 \quad M+1 \quad M+1]\begin{bmatrix}1\\1\\2\\1\\1\end{bmatrix} - (2M-2) = 4$$

$$z_7 - c_7 = \mathbf{wa_7} - c_7 = [0 \quad 0 \quad 0 \quad M+1 \quad M+1]\begin{bmatrix}0\\0\\0\\-1\\0\end{bmatrix} - (-M) = -1$$

$$z_8 - c_8 = \mathbf{wa_8} - c_8 = [0 \quad 0 \quad 0 \quad M+1 \quad M+1]\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} - 0 = M+1$$

$$z_9 - c_9 = \mathbf{wa_9} - c_9 = [0 \quad 0 \quad 0 \quad M+1 \quad M+1]\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} - 0 = M+1$$

x_7 enters the basis

$$\mathbf{y_7} = \mathbf{B^{-1}a_7} = \begin{bmatrix}1 & 0 & 0 & -1 & -1\\0 & 1 & 0 & -2 & 0\\0 & 0 & 1 & 1 & 1\\0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\0\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\2\\-1\\-1\\0\end{bmatrix} \qquad \begin{bmatrix}z_7 - c_7\\ \mathbf{y_7}\end{bmatrix} = \begin{bmatrix}-1\\1\\2\\-1\\-1\\0\end{bmatrix}$$

	BASIS INVERSE					RHS	x_7	Ratio
z_{big-M}	0	0	0	M+1	M+1	5	-1	
x_4	1	0	0	-1	-1	1	1	1
x_5	0	1	0	-2	0	3	2	1.5
x_6	0	0	1	1	1	9	-1	-
x_1	0	0	0	1	0	1	-1	-
x_3	0	0	0	0	1	4	0	-

x_4 leaves the basis

Iteration 5

	BASIS INVERSE					RHS
z_{big-M}	1	0	0	M	M	6
x_7	1	0	0	-1	-1	1
x_5	-2	1	0	0	2	1
x_6	1	0	1	0	0	10
x_1	1	0	0	0	-1	2
x_3	0	0	0	0	1	4

$\mathbf{w} = [1 \quad 0 \quad 0 \quad M \quad M]$

For non-basic variables calculate $z_j - c_j = \mathbf{wa_j} - \mathbf{c_j}$

$$z_2 - c_2 = \mathbf{wa_2} - c_2 = [1 \quad 0 \quad 0 \quad M \quad M]\begin{bmatrix}1\\1\\2\\1\\1\end{bmatrix} - (2M-2) = 3$$

$$z_4 - c_4 = \mathbf{wa_4} - c_4 = [1 \quad 0 \quad 0 \quad M \quad M]\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} - 0 = 1$$

$$z_8 - c_8 = \mathbf{wa_8} - c_8 = [1 \quad 0 \quad 0 \quad M \quad M]\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} - 0 = M$$

$$z_9 - c_9 = \mathbf{wa_9} - c_9 = [1 \quad 0 \quad 0 \quad M \quad M]\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} - 0 = M$$

Since $z_j - c_j > 0$ optimal solution is founded.

Optimal Solution:

$x_1{}^* = 2, \; x_2{}^* = 0, \; x_3{}^* = 4, \; x_4{}^* = 0, \; x_5{}^* = 1, \; x_6{}^* = 10, \; x_7{}^* = 1, \; x_8{}^* = 0, \; x_9{}^* = 0$ and $\mathrm{z}^* = 6$

Question 3

a)

i: warehouse index i=1,2,3

j:store index j=1,2, 3

k:sent period k=1,2

l:demand request period l=1,2

t:transportation type 1=truck 2=kindle

c_{klt}: unit cost that is requested in l th period and sent in k th period with t type of transportation per kms and number of books. ($/ km*number of books)

x_{ijklt}: number of comic books that are sent from i^{th} warehouse to j^{th} store requested in l th period and sent in k th period and transported with t type (number of books) (**Decision variable**)

y_{jl}: j th store's demand which is requested in l th period.(number of books)

z_{ij}: distance that will be spend while transporting the book from i th warehouse to j th store (kms)

a_{ik}: # of books that must be in the i th warehouse in period k (number of books)

min w=$\sum_{k,l,t}(c_{klt})\left(\sum_{i,j} x_{ijklt}\ z_{ij}\right)$(**Objective Function**)

a_{ik}=200.000

$c_{kl2} = 0.05$

$c_{kl1} = 0.1$

$z_{23} = 0$

$\sum_{j,l,t} x_{ijklt} \leq a_{ik} \quad \forall\, i,k$ (**Supply constraint**)

$\sum_{i,k,t} x_{ijklt} \geq y_{jl} \quad \forall j,l$ (**Demand constraint**)

Question 4.

a.

$$B=\begin{bmatrix} 3 & 2 & 1 & 1 \\ 0,4 & 0,8 & 0,5 & 0 \\ 0,2 & 0,3 & 0,1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix},\ b=\begin{bmatrix} 1000 \\ 300 \\ 100 \\ 500 \end{bmatrix}$$

If it is a basic feasible solution then;

Inverse of the B matrix can be retrieved and $X_B=B^{-1}*b>=0$ must be provided.

$$B^{-1}=\begin{bmatrix} 0 & -4 & 6 & 1,4 \\ 0 & 2 & 2 & -1,2 \\ 0 & 2 & -8 & 0,8 \\ 1 & 6 & -14 & -2,6 \end{bmatrix}, \qquad X_B=\begin{bmatrix} 0 & -4 & 6 & 1,4 \\ 0 & 2 & 2 & -1,2 \\ 0 & 2 & -8 & 0,8 \\ 1 & 6 & 14 & -2,6 \end{bmatrix}*\begin{bmatrix} 1000 \\ 300 \\ 100 \\ 500 \end{bmatrix}=\begin{bmatrix} 100 \\ 200 \\ 200 \\ 100 \end{bmatrix}$$

Since both conditions are provided we can say it is a basic feasible solution.

In order to show it is an optimal solution we should show that reduced cost ($(z_j - c_j)$ for all NBV are nonnegative. ($z_j = w * a_j$ and $w = C_B * B^{-1}$)

$C_{BV} = [40 \quad 150 \quad 30 \quad 0]$

w= $C_{BV} * B^{-1} = [40 \quad 150 \quad 30 \quad 0] * \begin{bmatrix} 0 & -4 & 6 & 1,4 \\ 0 & 2 & 2 & -1,2 \\ 0 & 2 & -8 & 0,8 \\ 1 & 6 & -14 & -2,6 \end{bmatrix} = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}]$

for s_2; $z_5 - c_5 = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}] * \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 0 = 200 > 0$

for a_3; $z_6 - c_6 = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}] * \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - (-M) = 300 + M > 0$

for e_4; $z_7 - c_7 = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}] * \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} - 0 = 100 > 0$

for a_4; $z_8 - c_8 = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}] * \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - (-M) = M - 100 > 0$

Since all reduced cost for NBV are nonnegative, BV=$\{x_1, x_2, x_3, s_1\}$ is a optimal solution.

b.

We must check the right hand values:

$$\bar{\mathrm{b}} = \mathrm{B}^{-1}\mathrm{b} = \begin{bmatrix} 0 & -4 & 6 & 1,4 \\ 0 & 2 & 2 & -1,2 \\ 0 & 2 & -8 & 0,8 \\ 1 & 6 & -14 & -2,6 \end{bmatrix} \begin{bmatrix} 1000 \\ 300 + \delta \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 100 - 4\delta \\ 200 + 2\delta \\ 200 + 2\delta \\ 100 + 6\delta \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

so $-16,67 \leq \delta \leq 25$

300+25=325

300-16,67=283,33

labor constraint should be between 283,33 and 325

c.

If the company's labor capacity is changed to 310 hours then the change will be 10 which is between $-16{,}67 \leq \delta \leq 25$ so that the optimal solution will not change.

But the total profit will change;

The new X_B values :

$$X_B=\begin{bmatrix}100-4\delta\\200+2\delta\\200+2\delta\\100+6\delta\end{bmatrix}=\begin{bmatrix}100-4*10\\200+2*10\\200+2*10\\100+6*10\end{bmatrix}=\begin{bmatrix}60\\220\\220\\160\end{bmatrix}$$

According to this max z will be 40x1+150x2+30x3= 42000 and three products will be produced.

d.

If the change is between $-16{,}67 \leq \delta \leq 25$ than we can say the optimum solution will remain same but in question it is "-20" which is out of the range. So that current solution will be change. We must check the right hand values

RHS values will be $B^{-1}b=\begin{bmatrix}180\\160\\160\\-20\end{bmatrix}$ We can apply dual simplex method.

$$z=C_BB^{-1}b=[40 \quad 150 \quad 30 \quad 0]\begin{bmatrix}180\\160\\160\\-20\end{bmatrix}=36000$$

z	x_1	x_2	x_3	s_1	s_2	a3	e4	a4	STD
1	0	0	0	0	200	300	100	-100	36000
0	1	0	0	1	-4	6	-1,4	1,4	180
0	0	1	0	0	2	2	-1,2	-1,2	160
0	0	0	1	0	2	-8	0,8	0,8	160
0	0	0	0	1	6	-14	-2,6	-2,6	-20

According to dual simplex tableau, a3 will enter the bases. Since the artificial variable enter the bases, we can say there is no feasible solution if we change the lobar constraints to the 280.

e.

Since x_2 is in bases C_{BV} will change. We must control reduced cost of all nonbasic variables whether they remain nonnegative.

C'_{BV}=[40 150 + δ 30 0]

$for\ s_2,$

$z_5 - c_5 = C'_{BV}B^{-1}a_5 - c_5 =$

$$[40 \quad 150+\delta \quad 30 \quad 0]\begin{bmatrix}0 & -4 & 6 & 1,4\\0 & 2 & 2 & -1,2\\0 & 2 & -8 & 0,8\\1 & 6 & -14 & -2,6\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} - 0 = 200 + 2\delta \geq 0 \Rightarrow \delta \geq -100$$

$for\ a_3,$

$z_6 - c_6 = C'_{BV}B^{-1}a_6 - c_6 =$

$$[40 \quad 150+\delta \quad 30 \quad 0]\begin{bmatrix}0 & -4 & 6 & 1,4\\0 & 2 & 2 & -1,2\\0 & 2 & -8 & 0,8\\1 & 6 & -14 & -2,6\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix} + M = 300 + 2\delta + M \geq 0 \Rightarrow \delta \geq (-150 - M)/2$$

$for\ e_4,$

$z_7 - c_7 = C'_{BV}B^{-1}a_7 - c_7 =$

$$[40 \quad 150+\delta \quad 30 \quad 0]\begin{bmatrix}0 & -4 & 6 & 1,4\\0 & 2 & 2 & -1,2\\0 & 2 & -8 & 0,8\\1 & 6 & -14 & -2,6\end{bmatrix}\begin{bmatrix}0\\0\\0\\-1\end{bmatrix} + 0 = 100 + 1,2\delta \geq 0 \Rightarrow \delta \geq -83,333$$

$for\ a_4,$

$z_8 - c_8 = C'_{BV}B^{-1}a_8 - c_8 =$

$$[40 \quad 150+\delta \quad 30 \quad 0]\begin{bmatrix}0 & -4 & 6 & 1,4\\0 & 2 & 2 & -1,2\\0 & 2 & -8 & 0,8\\1 & 6 & -14 & -2,6\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + M = -100 - 1,2\delta + M \geq 0 \Rightarrow (M + 100)/1,2 \geq \delta$$

$\frac{M+100}{1,2} = \infty \Rightarrow$ If δ is between $-83,33 \leq \delta \leq \infty$ then current solution will remain optimal.

$150 - 83,33 = 66,67$

If values of the sales price of a chair is between $66,67 \leq c_2 \leq \infty$ then the current solution will remain optimal.

f.

Since 100 tl is between $66,67 \leq c_2 \leq \infty$ so the current solution will remain optimal but the total profit will change:

$$z = C'_{BV}X_B = \begin{bmatrix} 40 & 100 & 30 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \\ 200 \\ 100 \end{bmatrix} = 40\ 000$$

g.

60 TL is not in allowable range. So that the base solution will change.

The reduced cost (z_j-c_j) for all NBV should be checked.

$$w = C_B B^{-1} = \begin{bmatrix} 40 & 60 & 30 & 0 \end{bmatrix} \begin{bmatrix} 0 & -4 & 6 & 1{,}4 \\ 0 & 2 & 2 & -1{,}2 \\ 0 & 2 & -8 & 0{,}8 \\ 1 & 6 & -14 & -2{,}6 \end{bmatrix} = \begin{bmatrix} 0 & 20 & 120 & 8 \end{bmatrix}$$

For s_2,

$z_5 - c_5 = wa_5 - c_5 =$

$$\begin{bmatrix} 0 & 20 & 120 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 0 = 20$$

$for\ a_3,$

$z_6 - c_6 = wa_6 - c_6 =$

$$\begin{bmatrix} 0 & 20 & 120 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + M = M + 120$$

$for\ e_4,$

$z_7 - c_7 = wa_7 - c_7 =$

$$\begin{bmatrix} 0 & 20 & 120 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} + 0 = -8$$

$for\ a_4,$

$z_8 - c_8 = C'_{BV}B^{-1}a_8 - c_8 =$

$$\begin{bmatrix} 0 & 20 & 120 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + M = M + 8$$

Since e4 has the most negative reduced cost it will enter the bases. By using the formula $B^{-1}a_7$, determine the y_7 and by using ratio test determine the leaving variables:

	0	20	120	8	22000
x_1	0	-4	6	1,4	100
x_2	0	2	2	1,2	200
x_3	0	2	-8	0,8	200
s_1	1	6	-14	-2,6	100

e4	oran
-8	
-1,4	-
1,2	166,07
-0,8	-
2,6	38,46

s_1 leaves the bases and the new table;

1. iteration

	3,08	38,46	76,92	0	22307,69
x_1	0,54	-0,77	-1,54	0	153,85
x_2	-0,46	-0,77	8,46	0	153,85
x_3	0,31	3,85	-12,31	0	230,77
e_4	0,38	2,31	-5,38	-1	38,46

Reduced Cost of NBV;

For s_1,

$$z_4 - c_4 = wa_4 - c_4 = [3{,}08 \quad 38{,}46 \quad 76{,}92 \quad 0]\begin{bmatrix}1\\0\\0\\0\end{bmatrix} - 0 = 3{,}07 > 0$$

$for\ s_2$,

$$z_5 - c_5 = wa_5 - c_5 = [3{,}08 \quad 38{,}46 \quad 76{,}92 \quad 0]\begin{bmatrix}0\\1\\0\\0\end{bmatrix} - 0 = 38{,}46 > 0$$

$for\ a_3$,

$$z_6 - c_6 = wa_6 - c_6 = [3{,}08 \quad 38{,}46 \quad 76{,}92 \quad 0]\begin{bmatrix}0\\0\\1\\0\end{bmatrix} + M = M + 76{,}92 > 0$$

$for\ a_4$,

$$z_8 - c_8 = wa_8 - c_8 = [3{,}08 \quad 38{,}46 \quad 76{,}92 \quad 0]\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + M = M > 0$$

Since all reduced cost are nonnegative this solution is a new optimal solution. New Z will be 22307,69 . x1 and x2 should be produced 153,85 units and x3 should be produced 230,76 units.

h.

c1=40+3Δ

c2=150+Δ

x_1 *and* x_2 arebasic variables, so C_{BV} will change. We must control reduced cost of all nonbasic variables wheher they remain nonnegative.

w=C_{BV}*B^{-1} = $[40+3\Delta \quad 150+\Delta \quad 30 \quad 0]\begin{bmatrix} 0 & -4 & 6 & 1{,}4 \\ 0 & 2 & 2 & -1{,}2 \\ 0 & 2 & -8 & 0{,}8 \\ 1 & 6 & -14 & -2{,}6 \end{bmatrix}$

=$[0 \quad 200-10\Delta \quad 300+20\Delta \quad -100+3\Delta]$

$for\ s_2,$

$$z_5-c_5=wa_5-c_5=[0 \quad 200-10\Delta \quad 300+20\Delta \quad -100+3\Delta]\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}-0$$

$$=200-10\Delta$$

200-10Δ≥0 → $\Delta \leq 20$

$for\ a_3,$

$$z_6-c_6=wa_6-c_6=[0 \quad 200-10\Delta \quad 300+20\Delta \quad -100+3\Delta]\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}+M$$

$$=300+20\Delta+\mathrm{M}$$

300+20Δ+M≥0 → $\Delta \geq (-\mathrm{M}+300)/20$

$for\ e_4,$

$$z_7-c_7=wa_7-c_7=[0 \quad 200-10\Delta \quad 300+20\Delta \quad -100+3\Delta]\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}-0$$

$$=100-3\Delta$$

100-3Δ≥0 → $\Delta \leq 33{,}33$

$for\ a_4,$

$$z_8 - c_8 = wa_8 - c_8 = [0 \quad 200 - 10\Delta \quad 300 + 20\Delta \quad -100 + 3\Delta] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + M$$

$$= -100 + 3\Delta + M$$

-100+3Δ+M≥0 → $\Delta \geq (-M + 100)/3$

If $\Delta \leq 20$ then the current solution will remain same.

i.

We must check the right hand values;

$$\bar{b} = B^{-1}b = \begin{bmatrix} 0 & -4 & 6 & 1{,}4 \\ 0 & 2 & 2 & -1{,}2 \\ 0 & 2 & -8 & 0{,}8 \\ 1 & 6 & -14 & -2{,}6 \end{bmatrix} \begin{bmatrix} 1000 + 20\Delta \\ 300 \\ 100 + \Delta \\ 500 \end{bmatrix} = \begin{bmatrix} 100 + 6\Delta \\ 200 + 2\Delta \\ 200 - 8\Delta \\ 100 + 6\Delta \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ should be provided.

If the Δ is between $-16{,}67 \leq \Delta \leq 25$ then the current optimum solution will remain same.

j.

Answer:

max z = $40x_1 + 150x_2 + 30x_3 + 70x_4$

s.t;
$3x_1 + 2x_2 + x_3 + 4x_4 =< 1000$
$0{,}4x_1 + 0{,}8x_2 + 0{,}5x_3 + 0{,}6x_4 =< 300$
$0{,}2x_1 + 0{,}3x_2 + 0{,}1x_3 + 0{,}2x_4 = 100$
$x_1 + x_2 + x_3 + x_4 >= 500$
$x_1, x_2, x_3 >= 0$

For new variables reduced cost must be nonnegative,

$$w = C_{BV}*B^{-1} = [40 \quad 150 \quad 30 \quad 0] * \begin{bmatrix} 0 & -4 & 6 & 1{,}4 \\ 0 & 2 & 2 & -1{,}2 \\ 0 & 2 & -8 & 0{,}8 \\ 1 & 6 & -14 & -2{,}6 \end{bmatrix} = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}]$$

reduced cost for x_4;

$$z_4 - c_4 = wa_j - c_j = [\mathbf{0} \quad \mathbf{200} \quad \mathbf{300} \quad \mathbf{-100}] * \begin{bmatrix} 4 \\ 0{,}6 \\ 0{,}2 \\ 1 \end{bmatrix} - 70 = 80 - 70 = 10 \geq 0$$

The current solution will remain same. If the sales price of new product is more than 80 TL then producing it will be profitable.